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Class 10
MATHEMATICS

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Previous Year CBSE Question Paper 2017 Solutions

CBSE BOARD EXAM PAPER ANSWER - 2017
Class 10 - Mathematics
Set – 1, Code - 30/1

Mathematics (Standard)
Previous Year Paper 2017

Max. Marks: 80 **Duration: 3 hrs.**

General Instructions:

a) All questions are compulsory

b) The question paper consists of 30 questions divided into four sections A, B, C & D.

c) Section A comprises of 4 questions of 1 mark each.

d) Section B comprises of 6 questions of 2 marks each.

e) Section C comprises of 10 questions of 3 marks each.

f) Section D comprises 11 questions of 4 marks each.

g) There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

h) Use of calculators is not permitted.

SECTION - A

Question 1:

What is the common difference of an AP in which $a_{21} - a_7 = 84$?

Answer:

Let a be the first term and d be the common difference of AP.

We know that n^{th} term of AP is given as

$a_n = a + (n - 1)d$

According to question,

$a_{21} - a_7 = 84$

=> $\{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$

=> $(a + 20d) - (a + 6d) = 84$

=> $a + 20d - a - 6d = 84$

=> $14d = 84$

=> $d = \frac{84}{14}$

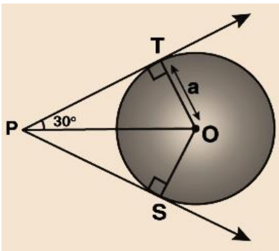
=> $d = 6$

So, the common difference of AP is 6.

Question 2:

If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is 60^0, then find the length of OP.

Answer:



From the figure,

PT and PS are two tangents of the circle

So, OT ⊥ PT and OS ⊥ PS

Also, given ∠TPS = 60°

So, $\angle TPO = \frac{60°}{2} = 30°$

Now, from right angled TPO,

$\sin 30° = \frac{OT}{OP}$

=> $\frac{1}{2} = \frac{a}{OP}$

=> OP = 2a

Question 3:

If a tower 30 m high, casts a shadow 10√3 m long on the ground, then what is the angle of elevation of the sun?

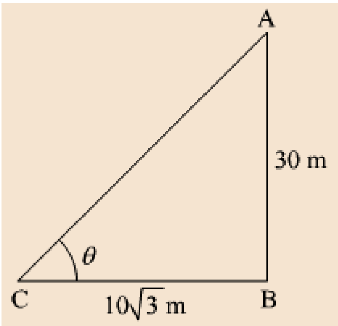
Answer:

Let AB be the tower and BC be the length of shadow of the tower as shown in the figure.

So, AB = 30 m and BC = 10√3 m

Again, let the angle of elevation of the sum from the ground be θ.

Now, in ΔABC,



$\tan θ = \frac{AB}{BC}$

=> $\tan θ = \frac{30}{10\sqrt{3}}$

=> $\tan θ = \frac{3}{\sqrt{3}}$

=> $\tan θ = \frac{\sqrt{3} * \sqrt{3}}{\sqrt{3}}$

=> tan θ = √3

=> $\tan θ = \tan 60^0$

=> $θ = 60^0$

Hence, the angle of elevation of the sum from the ground is 60^0.

Question 4:

The probability of selecting a rotten apple randomly from a heap of 900 apples is 0·18. What is the number of rotten apples in the heap?

Answer:

Let E be the event of selecting a rotten apple.

Again, let n be the number of rotten apples in the heap

Now, P(selecting a rotten apple) $= \frac{\text{number of rotten apples}}{\text{total number of apples}}$

=> $0.18 = \frac{n}{900}$

=> n = 0.18 * 900

=> n = 162

Thus, total number of rotten apples is 162.

SECTION – B

Question 5:

Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Answer:

Let first root of the quadratic equation is α.

Then the second root = 6α

Now, sum of roots = $\frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$

=> $\alpha + 6\alpha = -\frac{-14}{p}$

=> $7\alpha = \frac{14}{p}$

=> $\alpha = \frac{2}{p}$

Again, product of roots = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

=> $\alpha * 6\alpha = \frac{8}{p}$

=> $6\alpha^2 = \frac{8}{p}$

=> $6 * (\frac{2}{p})^2 = \frac{8}{p}$

=> $\frac{24}{p^2} = \frac{8}{p}$

=> p = 3

Hence, the value of p is 3.

Question 6:

Which term of the progression 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term?

Answer:

Given, progression is 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ...

First term, a = 20

Common difference, d = $19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{77-80}{4} = \frac{-3}{4}$

Let the n^th^ term of the progression be the first negative term.

So, $a_n < 0$

=> a + (n - 1)d < 0

=> $20 + (n - 1)\left(-\frac{3}{4}\right) < 0$

=> $20 - \frac{3n}{4} + \frac{3}{4} < 0$

=> $\frac{3n}{4} > 20 + \frac{3}{4}$

=> $\frac{3n}{4} > \frac{83}{4}$

=> 3n > 83

=> $n > \frac{83}{3}$

=> $n > 27\frac{2}{3}$

=> n = 28

So, the 28^th^ term of the progression is the first negative term.

Question 7:

Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Answer:

Let AB be a chord of a circle with centre O.

Again let AD and BD be the tangents on the circle at A and B respectively.

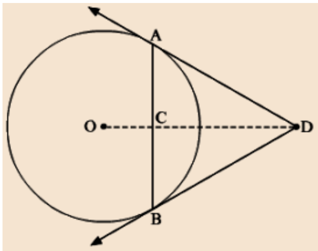
We know that the line segment joining the centre to the external point bisect the angle between two tangents.

So, ∠ADC = ∠BDC 1

Now, in ΔDCA and ΔDCB, we have

DA = DB [Tangents from an external point on the circle are equal]

∠ADC = ∠BDC [From equation 1]



DC = DC [Common]

So, ΔDCA ≅ ΔDCB [By SAS congruency rule]

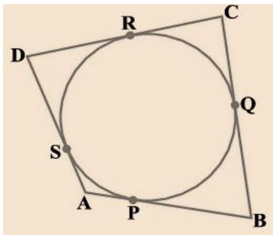
=> ∠DAC = ∠DBC [By CPCT]

Question 8:

A circle touches all the four sides of a quadrilateral ABCD. Prove that

AB + CD = BC + DA

Answer:



It can be observed that

DR = DS (Tangents on the circle from point D) .... (1)

CR = CQ (Tangents on the circle from point C) .... (2)

BP = BQ (Tangents on the circle from point B) .... (3)

AP = AS (Tangents on the circle from point A) .... (4)

Adding all these equations, we get

DR + CR + BP + AP = DS + CQ + BQ + AS

=> (DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

=> CD + AB = AD + BC

=> AB + CD = AD + BC

Question 9:

A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, then find the coordinates of P and Q.

Answer:

Let the line intersects y-axis at point P(0, y) and x-axis at Q(x, 0).

Given, (2, -5) is the mid-point of PQ.

Now, using mid-point formula, we get

=> $\left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$

=> $\left(\frac{x}{2}, \frac{y}{2}\right) = (2, -5)$

=> $\frac{x}{2} = 2$ and $\frac{y}{2} = -5$

=> x = 4, y = -10

So, the coordinates of points P and Q are (0, -10) and (4, 0) respectively.

Question 10:

If the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal, then prove that 3x = 2y.

Answer:

Given, the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal.

=> PA = PB

=> $\sqrt{(x - 5)^2 + (y - 1)^2} = \sqrt{(x + 1)^2 + (y - 5)^2}$

Squaring on both sides, we get

$(x - 5)^2 + (y - 1)^2 = (x + 1)^2 + (y - 5)^2$

=> $x^2 - 10x + 25 + y^2 - 2y + 1 = x^2 - 2x + 1 + y^2 - 10y + 25$

=> $x^2 - 10x + 25 + y^2 - 2y + 1 - x^2 + 2x - 1 - y^2 + 10y - 25 = 0$

=> $-12x + 8y = 0$

=> $12x = 8y$

=> $3x = 2y$

Hence, proved.

SECTION - C

Question 11:

If $ad \neq bc$, then prove that the equation

$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Answer:

Given, quadratic equation is $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$

We know that $D = b^2 - 4ac$

=> $D = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$

=> $D = 4[(a^2c^2 + b^2d^2 + 2abcd) - (a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)]$

=> $D = 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$

=> $D = 4(2abcd - a^2d^2 - b^2c^2)$

=> $D = -4(a^2d^2 + b^2c^2 - 2abcd)$

=> $D = -4(ad - bc)^2$ 1

Again, given $ad \neq bc$

=> $ad - bc \neq 0$

=> $(ad - bc)^2 > 0$

=> $-4(ad - bc)^2 < 0$

=> $D < 0$ [From equation 1]

Thus, the given equation has no real roots.

Question 12:

The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

Answer:

Given, the first term of AP, a = 5

Last term of AP, l = 45

Let the common difference of AP be d and total number of terms be n.

Now, the sum of AP = 400

=> $\left(\frac{n}{2}\right) * (a + l) = 400$

=> $\left(\frac{n}{2}\right) * (5 + 45) = 400$ (Given a = 5, l = 45)

=> $\frac{50 * n}{2} = 400$

=> 50 * n = 400*2

=> 50 * n = 800

=> n = $\frac{800}{50}$

=> n = 16

Again,

l = a + (n - 1)d

=> 45 = 5 + (16 - 1)d

=> 45 - 5 = 15d

=> 40 = 15d

=> d = $\frac{40}{15}$

=> d = $\frac{8}{3}$

So, the number of terms in AP is 16 and common difference is $\frac{8}{3}$.

Question 13:

On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower.

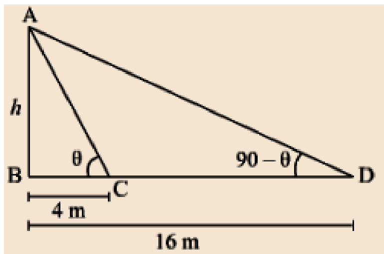
Answer:

Let AB be the tower of height h.

Again let the angle of elevation of the top of the tower from point C on the ground be θ.

Then the angle of elevation of the top of the tower from point D is 90^0 - θ.

From the figure, BC = 4 cm, BD = 16 cm



In ΔABC,

$\tan\theta = \frac{AB}{BC} = \frac{h}{4}$ 1

In ΔABD,

$\tan(90^0 - \theta) = \frac{AB}{BD}$

=> $\cot\theta = \frac{h}{16}$ 2

Multiplying equations 1 and 2, we get

$\tan\theta * \cot\theta = \left(\frac{h}{4}\right) * \left(\frac{h}{16}\right)$

=> $\frac{h^2}{64} = 1$

=> $h^2 = 64$

=> h = 8

So, the height of the tower is 8 m.

Question 14:

A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

Answer:

Let the number of black balls be x.

Given, number of white balls = 15

Total number of balls in the bag = x + 15

It is given that,

The probability of drawing a black ball from the bag is thrice that of drawing a white ball

=> P(black balls) = 3 * P(white balls)

=> $\frac{x}{x+15} = 3 * \frac{15}{x+15}$

=> x = 45

So, the number of black balls is 45.

Question 15:

In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P(2, -2) and Q(3, 7)? Also find the value of y.

Answer:

Let the point R$\left(\frac{24}{11}, y\right)$ divides the line PQ in the ratio k : 1.

By using section formula, we have

$$\left\{\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right\} = \left(\frac{24}{11}, y\right)$$

Equating x and y coordinates, we get

$$\frac{3k+2}{k+1} = \frac{24}{11} \text{and} \frac{7k-2}{k+1} = y$$

=> 11(3k + 2) = 24(k + 1) and (7k - 2) = y(k + 1)

=> 33k + 22 = 24k + 24 and 7k - 2 = yk + y

=> 33k – 24k = 24 - 22 and 7k - 2 = yk + y

=> 9k = 2 and 7k - 2 = yk + y

=> k = $\frac{2}{9}$ and 7k - 2 = yk + y

Therefore, the point R divides the line PQ in the ratio $\frac{2}{9}$: 1 i.e. 2 : 9

Pu k = $\frac{2}{9}$ in the equation 7k - 2 = yk + y, we get

=> 7 * $\frac{2}{9}$ - 2 = y * $\frac{2}{9}$ + y

=> $\frac{14}{9} - 2 = \frac{11y}{9}$

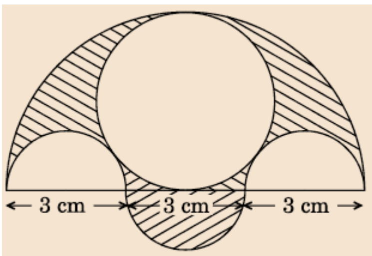
=> $\frac{-4}{9} = \frac{11y}{9}$

=> y = $\frac{-4}{11}$

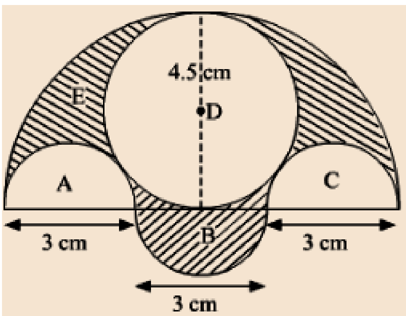
So, the coordinates of point R is $(\frac{24}{11}, \frac{-4}{11})$.

Question 16:

Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region.



Answer:



From the figure, we have

Area of shaded region = area of semicircle with diameter 9 cm - area of two semicircles with diameter 3 cm - area of circle with diameter 4.5 cm + area of semicircle with diameter 3 cm

=> Area of shaded region = area of semicircle with radius 4.5 cm – 2 * area of semicircles with radius 1.5 cm - area of circle with radius 2.25 cm + area of semicircle with radius 1.5 cm

=> Area of shaded region = $\frac{\pi * 4.5 * 4.5}{2} - 2 * \frac{\pi * 1.5 * 1.5}{2} - \pi * 2.25 * 2.25 + \frac{\pi * 1.5 * 1.5}{2}$

=> Area of shaded region = $\frac{\pi(20.25 - 2.25)}{2} - \pi * 5.0625$

=> Area of shaded region = 9π - 5.0625 π

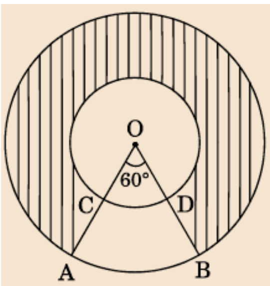
=> Area of shaded region = 3.9375π

=> Area of shaded region = $3.9375 * \left(\frac{22}{7}\right)$

=> Area of shaded region = 12.375 cm^2

Question 17:

In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If ∠AOB = 60^0, find the area of the shaded region. [Use π = $\frac{22}{7}$]



Answer:

Given, radius of inner circle, r = 21 cm

And radius of outer circle, R = 42 cm

Here, ∠AOB = 60°

So, area of the circular ring = $\pi R^2 - \pi r^2$

$= \pi[R^2 - r^2]$

$= \pi[42^2 - 21^2]$

Now, area of ACDB = area of sector AOB – area of sector COD

$= \left(\frac{60^0}{360^0}\right) * \pi * R^2 - \left(\frac{60^0}{360^0}\right) * \pi * r^2$

$= \left(\frac{60^0}{360^0}\right) * \pi * [R^2 - r^2]$

$= \left(\frac{1}{6}\right) * \pi * [42^2 - 21^2]$

So, area of shaded region = area of circular ring – area of ACDB

$= \pi * [42^2 - 21^2] - \left(\frac{1}{6}\right) * \pi * [42^2 - 21^2]$

$= \pi * [42^2 - 21^2]\left(1 - \frac{1}{6}\right)$

$= \pi * (42 - 21)(42 + 21) * \left(\frac{5}{6}\right)$

$= \left(\frac{22}{7}\right) * 21 * 63 * \left(\frac{5}{6}\right)$

$= 3465 \text{ cm}^2$

Question 18:

Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 $\frac{\text{km}}{\text{hour}}$. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation?

Answer:

Given, width of the canal = 5.4 m

And depth of the canal = 1.8 m

Height of the standing water needed for irrigation = 10 cm = $\frac{10}{100}$ m = 0.1 m

Given, speed of the flowing water 25 $\frac{\text{km}}{\text{hr}} = \frac{25 * 1000}{60} \frac{\text{m}}{\text{min}}$

$= \frac{1250}{3} \frac{m}{min}$

Now, volume of the water flowing out of the canal in 1 minute

= area of opening canal * $\left(\frac{1250}{3}\right)$

= 5.4 * 1.8 * $\left(\frac{1250}{3}\right)$

= 4050 m^3

So, volume of water flowing out of the canal in 40 minutes = 40 * 4050 = 162000 m^3

Area of irrigation = $\frac{\text{Volume of water flowing out of the canal in 40 minutes}}{\text{Height of the standing water needed for irrigation}}$

$= \frac{162000}{0.1}$

= 1620000 m^2

= 162 hectare [Since 1 hectare = 10000 m^2]

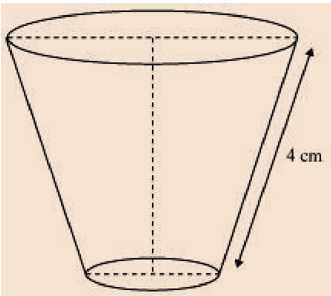
Hence, the area irrigated in 40 minutes is 162 hectare.

Question 19:

The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Answer:

Perimeter of upper circular end of frustum = 18 cm



=> $2\pi r_1 = 18$

=> $r_1 = \frac{18}{\pi}$

Perimeter of lower end of frustum = 6 cm

=> $2\pi r_2 = 6$

=> $r_2 = \frac{3}{\pi}$

Slant height (l) of frustum = 4 cm

CSA of frustum = $\pi(r_1 + r_2)l$

$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) * 4$

$= 12 * 4$

$= 48\ cm^2$

Therefore, the curved surface area of the frustum is $48\ cm^2$.

Question 20:

The dimensions of a solid iron cuboid are 4·4 m * 2·6 m * 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

Answer:

Let the length of pipe be h cm.

Now, volume of solid iron cuboid = 4·4 m * 2·6 m * 1·0 m

= 440 cm * 260 cm * 100 cm [1 m = 100 cm]

Given, internal radius of the pipe, r = 30 cm

External radius of the pipe, R = 30 + 5 = 35 cm

Now, volume of iron in the pipe = $\pi R^2h - \pi r^2h$

$= \pi h(R^2 - r^2)$

$= \pi h(35^2 - 30^2)$

$= \pi h(35 - 30)(35 + 30)$

$= \pi h * 5 * 65$

Volume of iron in the pipe = volume of solid iron cuboid

=> $\pi h * 5 * 65 = 440 * 260 * 100$

=> $h = \frac{440 * 260 * 1000}{\pi * 5 * 65}$

=> $h = \frac{440 * 260 * 1000 * 7}{22 * 5 * 65}$

=> h = 11200 cm

=> h = $\frac{11200}{100}$ m

=> h = 112 m

So, the length of pipe is 112 m.

SECTION - D

Question 21:

Solve for x: $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, x \neq -1, \frac{-1}{5}, -4$

Answer:

Given, $\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}$

=> $\frac{5x+1+3x+3}{(x+1)(5x+1)} = \frac{5}{x+4}$

=> $\frac{8x+4}{(x+1)(5x+1)} = \frac{5}{x+4}$

=> (8x + 4)(x + 4) = 5(x + 1)(5x + 1)

=> $8x^2 + 32x + 4x + 16 = 5(5x^2 + x + 5x + 1)$

=> $8x^2 + 36x + 16 = 5(5x^2 + 6x + 1)$

=> $8x^2 + 36x + 16 = 25x^2 + 30x + 5$

=> $25x^2 + 30x + 5 - 8x^2 - 36x - 16 = 0$

=> $17x^2 - 6x - 11 = 0$

=> $17x^2 - 17x + 11x - 11 = 0$

=> 17x(x - 1) + 11(x - 1) = 0

=> (x - 1)(17x + 11) = 0

=> x = 1, $\frac{-11}{17}$

Question 22:

Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank?

Answer:

Let the first pipe fills the tank in x hours.

So, another pipe can fill the tank in (x + 3) hours

Now, part of the tank filled by the first tank in 1 hour = $\frac{1}{x}$

And part of the tank filled by another tank in 1 hour = $\frac{1}{x+3}$

So, part of the tank filled by both the pipes in 1 hour = $\frac{1}{x} + \frac{1}{x+3}$

Given, two taps running together can fill a tank in $3\frac{1}{13} = \frac{40}{13}$ hours

Now, $\frac{1}{x} + \frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

=> $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$

=> $\frac{x+3+x}{x(x+3)} = \frac{13}{40}$

=> $\frac{2x+3}{x^2+3x} = \frac{13}{40}$

=> $40(2x + 3) = 13(x^2 + 3x)$

=> $80x + 120 = 13x^2 + 39x$

=> $13x^2 + 39x – 80x – 120 = 0$

=> $13x^2 - 41x – 120 = 0$

=> $13x^2 - 65x + 24 – 120 = 0$

=> $13x(x - 5) + 24(x - 5) = 0$

=> $(x - 5)(13 + 24) = 0$

=> $x = 5, \frac{-13}{24}$

Since time cannot be negative.

So, x = 5

Hence, Let the first pipe fills the tank in 5 hours.

And another pipe can fill the tank in (5 + 3) i.e. 8 hours.

Question 23:

If the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27), then find the ratio of their 9^{th} terms.

Answer:

Let a_1, a_2 be the first terms and d_1, d_2 be the common differences of the given two A.Ps.

Now, sum of their n terms are given as

$S_n = \left(\frac{n}{2}\right)\{2a_1 + (n - 1)d_1\}$ and $S'_n = \left(\frac{n}{2}\right)\{2a_2 + (n - 1)d_2\}$

Given that the ratio of the sum of the first n terms of two A.Ps is (7n + 1) : (4n + 27)

=> $S_n : S'_n = (7n + 1) : (4n + 27)$

=> $\frac{S_n}{S'_n} = \frac{7n+1}{4n+27}$

=> $\frac{\left(\frac{n}{2}\right)(2a_1 + (n-1)d_1)}{\left(\frac{n}{2}\right)(2a_2 + (n-1)d_2)} = \frac{7n+1}{4n+27}$

=> $\frac{(2a_1 + (n-1)d_1)}{(2a_2 + (n-1)d_2)} = \frac{7n+1}{4n+27}$

=> $\frac{\left(a_1 + \frac{(n-1)d_1}{2}\right)}{\left(a_2 + \frac{(n-1)d_2}{2}\right)} = \frac{7n+1}{4n+27}$ 1

Now, $a_9 = a + (9 - 1)d = a + 8d$

Compare it with $a + \left\{\frac{n-1}{2}\right\}d$, we get

$\frac{n-1}{2} = 8$

=> n = 17

Now, $\frac{a_9}{a'_9} = \frac{\left(a_1 + \frac{(17-1)d_1}{2}\right)}{\left(a_2 + \frac{(17-1)d_2}{2}\right)} = \frac{7*17+1}{4*17+27}$

=> $\frac{a_9}{a'_9} = \frac{(a_1 + 16d_1)}{(a_2 + 16d_2)} = \frac{120}{95}$

So, the ratio of 9th term of APs is 120 : 95

Question 24:

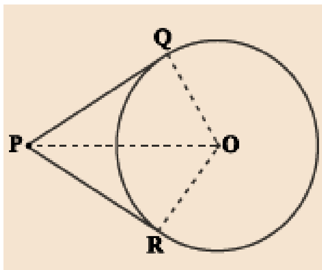
Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Answer:

We are given a circle with centre O, a point P lying outside the circle and two tangents PQ, PR on the circle from P as shown in the figure. We are required to prove that PQ = PR.

For this, we join OP, OQ and OR.

Then ∠OQP and ∠ORP are right angles, because these are angles between the radii and tangents, and since the tangent at any point of a circle is perpendicular to the radius through the point of contact, so, they are right angles.



Now in right triangles OQP and ORP,

OQ = OR (Radii of the same circle)

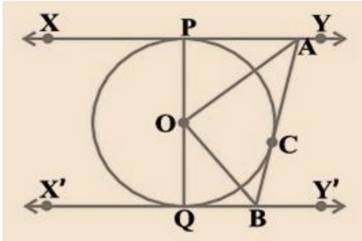
OP = OP (Common)

Therefore, Δ OQP ≅ Δ ORP (RHS)

This gives PQ = PR (CPCT)

Question 25:

In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and X'Y' at B. Prove that ∠AOB = 90^0.



Answer:

Let us join point O to C.

In ΔOPA and ΔOCA,

OP = OC [radii of the same circle]

AP = AC [tangents from point A]

AO = AO [Common side]

ΔOPA ≅ ΔOCA [SSS Congruence criterion]

∠POA = ∠COA ………..1

Similarly,

ΔOQB ≅ ΔOCB

∠QOB = ∠COB ………..2

Since POQ is a diameter of the circle, it is a straight line.

So, $\angle POA + \angle COA + \angle QOB + \angle COB = 180^0$

From equation 1 and 2, we get

=> $2\angle COA + 2\angle COB = 180^0$

=> $2(\angle COA + \angle COB) = 180^0$

=> $\angle COA + \angle COB = 90^0$

=> $\angle AOB = 90^0$

Question 26:

Construct a triangle ABC with side BC = 7 cm, $\angle B = 45^0$, $\angle A = 105^0$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC.

Answer:

Step of Construction:

1. Draw a line BC = 7 cm.

2. From point B, construct ∠CBX = 45^0 and from point C, draw ∠BCY = 180^0 – (105^0 + 45^0) = 30^0

3. The point of intersection of BX and CY gives A. Thus, ΔABC is obtained.

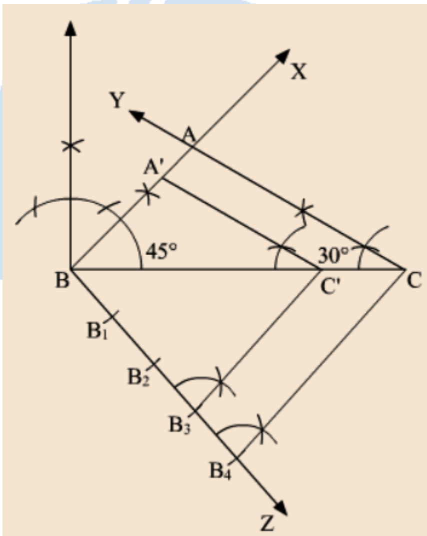
4. Draw any ray BZ making an acute angle with BC on the side opposite to the vertex A.

5. Locate four points B_1, B_2, B_3 and B_4 on BZ such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

6. Join B_4C and draw a line through B_3 parallel to B_4C that intersect BC at C'.

7. Draw a line through C' parallel to the line CA that intersects BA at A'.

Here, ΔA'BC' is the required triangle similar to ΔABC.



Question 27:

An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45^0 and 60^0 respectively. Find the width of the river. [Use √3 = 1·732]

Answer:

Let CD be the height of the aeroplane above the river at some instant.

Again let A and B be two points on both banks of the river in opposite directions.

Given, the height of the aeroplane above the river i.e. CD = 300 m

From the figure,

∠CAD = ∠ADX = 60^0 (Alternate angles)

∠CBD = ∠BDY = 45^0 (Alternate angles)

Now, in ΔACD,

$$\tan 60^0 = \frac{CD}{AC}$$

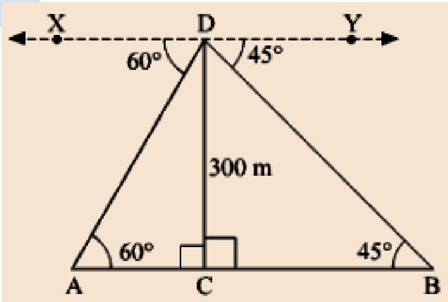
=> $\sqrt{3} = \frac{300}{AC}$

=> $AC = \frac{300}{\sqrt{3}}$

=> $AC = \left(\frac{300}{\sqrt{3}}\right) * \left(\frac{\sqrt{3}}{\sqrt{3}}\right)$

=> $AC = \frac{300\sqrt{3}}{3}$

=> AC = $100\sqrt{3}$



Again, in ΔBCD,

$$\tan 45^0 = \frac{CD}{BC}$$

=> $1 = \frac{300}{BC}$

=> BC = 300

Now, width of the river = BC + AC

= $300 + 100\sqrt{3}$

= 300 + 100 * 1.732

= 300 + 173.2

= 473.2

Hence, the width of the river is 473.2 m

Question 28:

If the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k - 1, 5k) are collinear, then find the value of k.

Answer:

Let us assume that the points $A(x_1, y_1) = (k + 1, 2k)$, $B(x_2, y_2) = (3k, 2k + 3)$ and $C(x_3, y_3) = (5k + 1, 5k)$ form a triangle.

Now, area(ΔABC) with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by

$\text{Area}(\Delta ABC) = \left(\frac{1}{2}\right)[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\text{Area}(\Delta ABC) = \left(\frac{1}{2}\right)[(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k + 1)(2k - 2k - 3)]$

$\text{Area}(\Delta ABC) = 2k^2 - 5k + 2$

Since, A, B, C are collinear

So, Area(ΔABC) = 0

=> $2k^2 - 5k + 2 = 0$

=> $(k - 2)(2k - 1) = 0$

=> $k = 2, \frac{1}{2}$

Question 29:

Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and (ii) even product.

Answer:

When two dice are tossed, then possible outcome are:

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6) {(x, y): x for 1st dice and y for 2nd dice}

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

So, total possible outcomes, n(S) = 36

(i) Let A be the event of getting an even sum

=> A = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}

So, total favourable outcome, n(A) = 18

Now, required probability $P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

(ii) Let B be the event of getting an even product

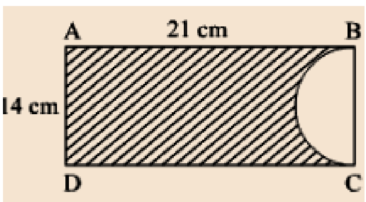
=> B = {(1, 1), (1, 2), (1, 5), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4,6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

So, total favourable outcome, n(B) = 24

Now, required probability $P(B) = \frac{n(B)}{n(S)} = \frac{24}{36} = \frac{2}{3}$

Question 30:

In the given figure, ABCD is a rectangle of dimensions 21 cm * 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



Answer:

Radius of semi-circle = $\frac{BC}{2} = \frac{14}{2}$ = 7 cm

From the figure,

Area of shaded region = Area of rectangle – area of semi-circle

= 21 * 14 – ($\frac{1}{2}$ * π * 7^2)

= 294 – $\left(\frac{1}{2} * \frac{22}{7} * 49\right)$

= 294 – 77

= 217 cm^2

Again, length of the boundary (perimeter) of shaded region

= AB + AD + DC + are BC

= 21 + 14 + 21 + ($\frac{1}{2}$ * 2π * 7)

= 56 + $\left(\frac{1}{2} * 2 * \frac{22}{7} * 7\right)$

= 56 + 22

= 77 cm

Question 31:

In a rain-water harvesting system, the rain-water from a roof of 22 m * 20 m drains into a cylindrical tank having diameter of base 2 m and height 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

Answer:

Given, length of the roof, l = 22 m

Width of the roof, b = 20 cm

Base radius of the cylindrical vessel, R = $\frac{2}{2}$ = 1m

Height of the cylindrical vessel, H = 3.5 m

Let height of the rainfall be h.

Now, total amount of rainfall = Volume of rain water collected in the cylindrical vessel

=> l * b * h = $\pi R^2 H$

=> 22 * 20 * h = $\frac{22}{7}$ * 1^2 * 3.5

=> 440h = 22 * 0.5

=> 440h = 11

=> h = $\frac{11}{440}$

=> h = 0.025 m

=> h = 0.025 * 100 cm

=> h = 2.5 cm

It is important to conserve the water for sustainable development. There are many different methods that can be used for conservation of water. One such method is rain water harvesting which not only avoids wastage of water but also helps in meeting the water demand during summer since much water is needed in this season.

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